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AN ANALYTICAL METHOD FOR THE PREDICTION
OF PRESSURE LAG INHERENT IN
BALLISTIC MISSILE PRESSURE SENSING SYSTEMS
WHEN SUBJECTED TO IMPULSE-TYPE
PRESSURE FUNCTIONS

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# LIST OF SYMBOLS

| | |
|---|---|
| $B$ | Constant |
| $C_1$ | Function of time |
| $C_2$ | Function of time |
| $D$ | Inside diameter of tubing |
| $K$ | System parameter |
| $L$ | Length of tubing |
| $m$ | Mass of air in system |
| $m'$ | Rate of mass flow |
| $P(t)$ | Transient input pressure |
| $P_1$ | Transient response pressure |
| $p$ | Pressure |
| $\bar{p}$ | Mean pressure in system |
| $p_i$ | Transient input pressure |
| $p_r$ | Transient response pressure |
| $\Delta p$ | Pressure difference or pressure lag |
| $Q$ | Rate of volume flow |
| $R$ | Gas constant |
| $T$ | Absolute temperature |
| $t$ | Time |
| $x$ | Spatial co-ordinate |
| $V_s$ | Internal volume of sensing element |
| $V_t$ | Internal volume of tubing |

| | |
|---|---|
| $V$ | Total internal volume of system $= V_s + V_t$ |
| $\lambda$ | Time lag constant |
| $\mu$ | Coefficient of viscosity |
| $\rho$ | Mass density of air |
| $\phi$ | Dimensionless geometric parameter $\left(= \frac{LV}{10^4 D^4}\right)$ |

In Appendix B, the following symbols were used to denote functions of the variable $t$

$$u,\ y,\ p,\ q,\ r,\ F,\ P,\ Q,\ R.$$

## SUMMARY

This study presents a method for the prediction of the pressure lag in missile pressure sensing systems during ascending flight. The analysis is based on the assumption of quasi-steady, isothermal, fully-developed laminar flow through constant area tubing. The equation derived for the response pressure is a non-linear, first order differential equation containing a parameter which is a function of the system geometry only, and independent of the dynamics of the flow. Comparison with available experimental data for the response of a system to inputs analagous to those encountered by multi-stage missiles in ascending flight indicates that the response pressure equation satisfactorily describes the behavior of the system.

# CHAPTER I

## INTRODUCTION

The last decade has seen significant changes in the nature of the problems associated with the prediction of the response, or the lag of the pressure measuring instrumentation in flying vehicles. This lag is created during diving or climbing flight or in the course of any flight maneuver where the input to the pressure measuring instrumentation is of a transient nature. Early studies[1,2,3*] were directed mainly towards fairly low speed airplanes and, as the pressure transients associated with the maneuvers of this class of airplane are small in nature, a linear treatment was found to be satisfactory. Use of this method results in a linear differential equation for the response pressure of the form

$$\lambda \frac{dP_1}{dt} + P_1 = P(t)$$

where $\lambda$ is a time constant, $P_1$ is the response pressure and $P(t)$ is the input pressure function.

However, a series of tests[4] conducted on the response of a system to a step input made it apparent that the "time constant" was, in fact, dependent on the size of the applied step input. This led Vaughn[4] to realize that, whatever the cause of this variation in the time constant, a linear treatment could no longer be used. He developed a non-linear

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*Numbered superscripts refer to the references in the bibliography.

treatment based upon the mass flow through a sharp-edged orifice attached to a pressure sensing instrument.

This method of attack leads to the non-linear differential equation

$$\frac{dP_1}{dt} = B \frac{T_1}{T} \sqrt{\left| P^2 - P_1^{\,2} \right|}$$

where $P_1$ is the response pressure, $P$ the transient input pressure, $T_1$ is the absolute temperature in the sensing volume, $T$ is the absolute input temperature and $B$ is a function of the system geometry and of the input temperature. This equation appears to give satisfactory results for systems with short line lengths subjected to small step inputs.

Recent advances in missile performance, particularly increases in rates of ascent and descent, have resulted in pressure input functions of a highly transient nature. Further, the internal geometry of most missiles requires a fairly long length of tubing from the orifice in the surface of the missile to the sensing instrument. In these circumstances the use of Vaughn's equation appears to give doubtful correlation of theory and experiment.

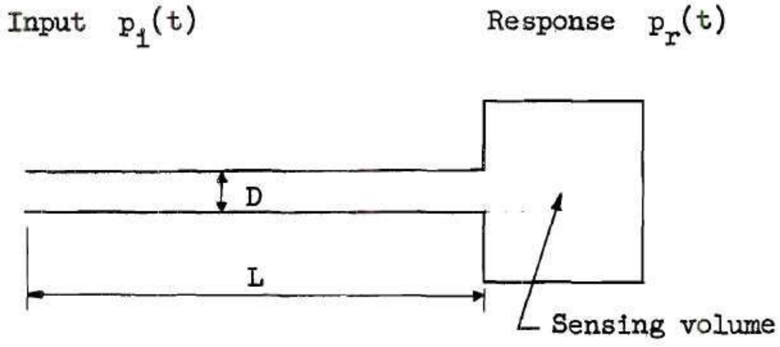
A further consideration is that a typical flight of a missile may well result in a pressure input which ranges from sea level atmospheric pressure to near vacuum in a short space of time. It therefore appears certain that a successful treatment of the problem must take account of the compressibility of the air. Also, as the time rate of change of input pressure may be large, a solution based on successive applications of "small-perturbation" theory will be of doubtful value.

The purpose of this study is to find a method suitable for the prediction of the pressure lag in missile instrumentation, taking into account the considerations mentioned above.

# CHAPTER II

## THEORY

Introductory remarks.--A schematic of the simple system considered in this analysis is shown below.




It consists of a length of tubing of diameter D and length L connected to a sensing instrument of internal volume $V_s$. (This, of course, is a considerably simpler system than that which would be found in any practical installation with its almost inevitable joints, elbows, etc.)

This analysis is based upon the assumptions of continuum flow, circular tubing of constant cross-sectional area, and fully developed laminar flow over the entire length of tubing. Changes of state of the fluid medium are assumed to take place according to the isothermal law. The justification for isothermal flow is based on the argument that the large ratio of internal surface area of the system to internal volume of the system, together with the large ratio of mass of metal in the system to mass of air in the system, make the instrument itself a relatively large capacity heat reservoir.

A quasi-steady solution, based on the supposition that the mass flow past any cross-section of the tubing is independent of the spatial position of that section within the system and dependent only upon time, is developed. The Hagen-Poiseuille Law for steady, incompressible, fully-developed laminar flow, with compressibility introduced through the equation of state, is used to determine the mass flow. Finally, use is made of the isothermal relation to relate the rate of change of mass in the system to the rate of change of response pressure. The equation resulting from the subsequent elimination of mass flow is a non-linear, first order differential equation for the response pressure.

<u>Derivation of the response pressure equation</u>.--

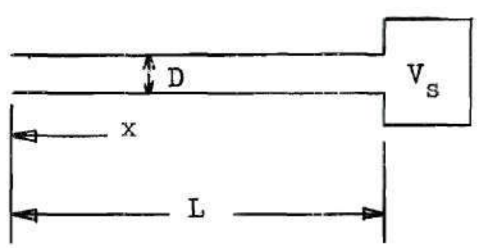



Figure 1.

If $m'$ and $Q$ are respectively the rates of mass flow and volumetric flow past a point in the system, then the Hagen-Poiseuille Law[5] for steady, incompressible, fully-developed laminar flow states that

$$m' = \rho Q = -\frac{\pi D^4}{128\mu}\,\rho\,\frac{\partial p}{\partial x} \tag{1}$$

where

D is the diameter of the tubing,

$\rho$ is the mass density,

p is the pressure,

$\mu$ is the coefficient of viscosity, and

x is the spatial co-ordinate, measured along the axis of the tube.

Strictly, this equation is applicable only to the flow of an incompressible fluid. However, it is well known[6] that this equation is valid for the steady, laminar, fully-developed flow of real fluids, in particular, fluids whose pressure, density and temperature satisfy an equation of state of the form $p = \rho RT$, where R is the gas constant and T the absolute temperature. It would therefore seem reasonable to expect that equation (1) could be modified to include the effects of compressibility by substituting for $\rho$ from the equation of state. Thus,

$$m' = \rho Q = -\frac{\pi D^4}{128\mu RT}\,p\,\frac{\partial p}{\partial x} \tag{2}$$

which can be rewritten in the form

$$m' = \rho Q = -\frac{\pi D^4}{128\mu RT}\,\frac{\partial}{\partial x}\left(\frac{p^2}{2}\right) \tag{3}$$

Now, the equation of continuity for a compressible flow of the quasi-steady nature supposed may be written

$$\frac{\partial}{\partial x}(\rho Q) = 0 \tag{4}$$

Substitution of equation (3) into equation (4) then yields

$$\frac{\partial^2}{\partial x^2}(p^2) = 0 \tag{5}$$

which can be integrated immediately to give

$$p^2 = C_1 x + C_2 \tag{6}$$

where $C_1$ and $C_2$ are functions of time, since equation (5) is a partial differential equation. The boundary conditions on $p$ are

$$\begin{aligned} &(a) \text{ at } x = 0, \quad p = p_i \\ &(b) \text{ at } x = L, \quad p = p_r \end{aligned} \tag{7}$$

Insertion of the boundary conditions (7) into equation (6) results in

$$p^2 = (p_r^2 - p_i^2)\frac{x}{L} + p_i^2 \tag{8}$$

which can be differentiated once with respect to $x$ to give

$$\frac{\partial}{\partial x}(p^2) = \frac{p_r^2 - p_i^2}{L} \tag{9}$$

Equation (9) may now be substituted into equation (3), yielding

$$m' = \rho Q = - \frac{\pi D^4}{256\ \mu RTL}(p_r^2 - p_i^2) \tag{10}$$

The equation of state for the air in the system may be written

$$\bar{p}\, V = mRT \tag{11}$$

where

$$\bar{p} = \frac{1}{V}\int p\,dV = \text{mean pressure in the system} \tag{12}$$

$V$ = internal volume of the system

$m$ = mass of air in the system

Assuming that changes of state of the air in the system take place isothermally, differentiation of equation (11) with respect to time yields

$$\frac{dm}{dt} = m' = \frac{V}{RT}\frac{d\bar{p}}{dt} \tag{13}$$

Eliminating m' between equation (10) and equation (13) yields

$$\frac{d\bar{p}}{dt} = -\frac{\pi D^4}{256\,\mu VL}\left(p_r^2 - p_i^2\right) \tag{14}$$

A further simplification can be effected by replacing $\frac{d\bar{p}}{dt}$ in equation (14) by $\frac{dp_r}{dt}$, so that

$$\frac{dp_r}{dt} = -\frac{\pi D^4}{256\,\mu VL}\left(p_r^2 - p_i^2\right) \tag{15}$$

This step can be justified qualitatively in the following way:

The pressure distribution in the tubing as given by equation (8) is such that the square of the pressure varies linearly with distance from the input end of the tube so that the mean pressure in the tubing is closer to $p_r$ than to $p_i$. Also, the pressure in the sensing volume is essentially equal to $p_r$ throughout. Both of these circumstances tend to weight the mean pressure in the system towards the response pressure, $p_r$. A more quantitative discussion of this approximation, involving the explicit evaluation of $\bar{p}$, is given in Appendix A.

Returning now to equation (15), if we define a "system parameter" K by

$$K = \frac{\pi D^4}{256 \mu V L} \tag{16}$$

then equation (15) can be rewritten as

$$\frac{dp_r}{dt} = K(p_i^2 - p_r^2) \tag{17}$$

Equation (17) is the differential equation for the response pressure, $p_r$, of the system for a given input pressure, $p_i(t)$.

Equation (17) is a particular case of Riccati's generalised equation. This equation is discussed, and functions $p_i(t)$ for which equation (17) has a closed form solution are presented in Appendix B.

# CHAPTER III

## COMPARISON OF THEORY AND EXPERIMENT

Concurrently with this study, Kowalsky[7] conducted a series of experimental tests to determine the response of typical pressure measuring systems to inputs analagous to those experienced by multi-stage missiles in ascending flight. Kowalsky measured the response of twelve different systems to four different input functions (shown in Fig. 2). The twelve systems comprised all combinations of four tubing lengths (30, 45, 60 and 75 inches) with three nominal tubing inside diameters (1/16, 1/8, and 5/32 - inch). Only one sensing volume of 1.7 $in.^3$ was tested. The discontinuities in the slope of the input functions are attributable to the particular experimental set-up employed by Kowalsky, but probably approximate the results of the firing of successive stages of a multi-stage missile.

Since Kowalsky's data consist of sets of values of the input pressure $p_i$ and the pressure lag $\Delta p = p_r - p_i$, comparison of these data with the theory of this paper is facilitated by rewriting equation (17) in terms of the quantities $p_i$ and $\Delta p$. Thus

$$\begin{aligned} \frac{dp_r}{dt} &= K\,(p_i^2 - p_r^2) \\ &= K\,(p_i + p_r)(p_i - p_r) \\ &= -K\,\Delta p(2p_i + \Delta p) \end{aligned}$$

so that

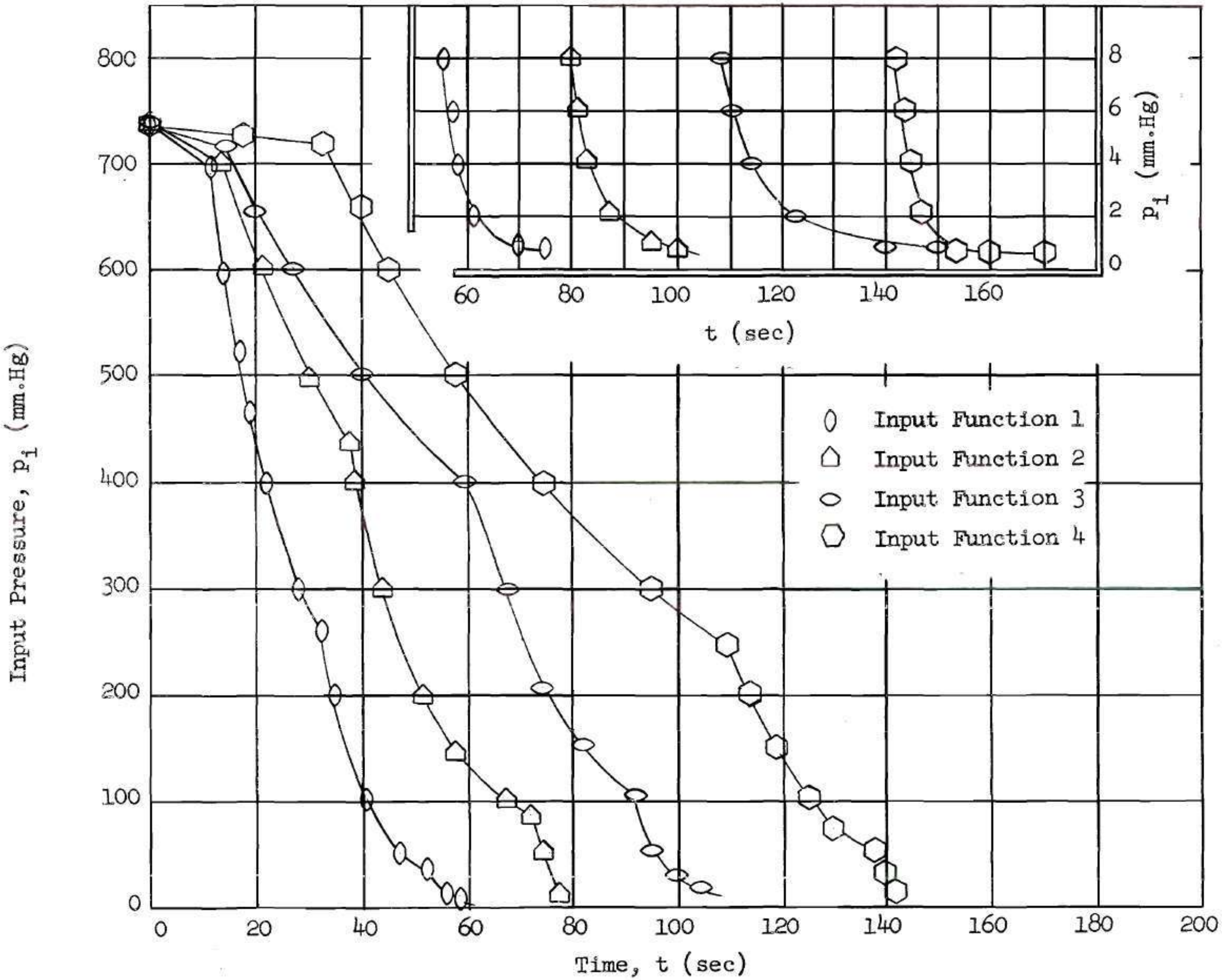



Fig. 2 Pressure Input Functions

$$\frac{d}{dt}(p_r - p_i) = \frac{d}{dt}(\Delta p)$$
$$= -K\Delta p(2p_i + \Delta p) - \frac{dp_i}{dt}$$

or, using a prime to denote differentiation with respect to time

$$\Delta p' = -K\Delta p(2p_i + \Delta p) - p_i' \qquad (18)$$

Kowalsky's tests were run under conditions of little variation in ambient temperature so that $\mu$ may be assigned a constant value in equation (16). The value chosen was that corresponding to a temperature of 75°F. With this value equation (16) gives

$$K = \frac{8.99}{\phi}\left(\frac{1}{\text{mm.Hg. sec}}\right)$$

where $\phi$ is the dimensionless geometric parameter $\frac{LV}{10^4 D^4}$. The quantity $\frac{LV}{D^4}$ appears in equation (16), the factor of $10^4$ being introduced for numerical convenience only. It is worthy of note that the quantity $K\mu$ is non-dimensional and has the value $0.01227 \frac{D^4}{LV}$.

The labor involved in computing solutions of equation (18) is considerable so the following procedure was adopted. Four representative solutions of equation (18) were computed using the experimental data for $p_i$. These solutions are presented in Figs. 3,4,5,6 together with the corresponding experimental data. The agreement can be seen to be definitely acceptable.

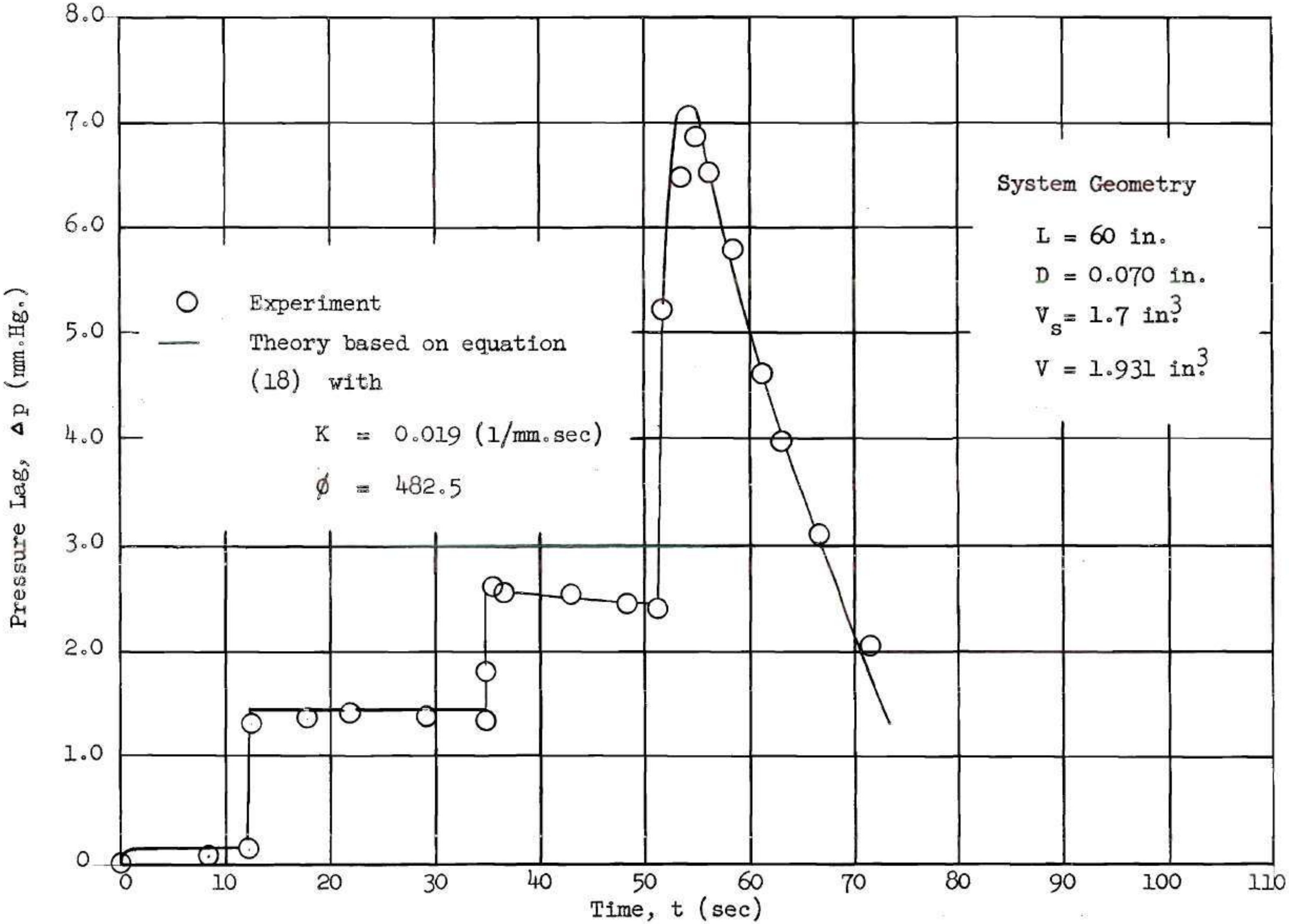



Fig. 3 Correlation of Experiment with Theory for Input Pressure Function No. 1

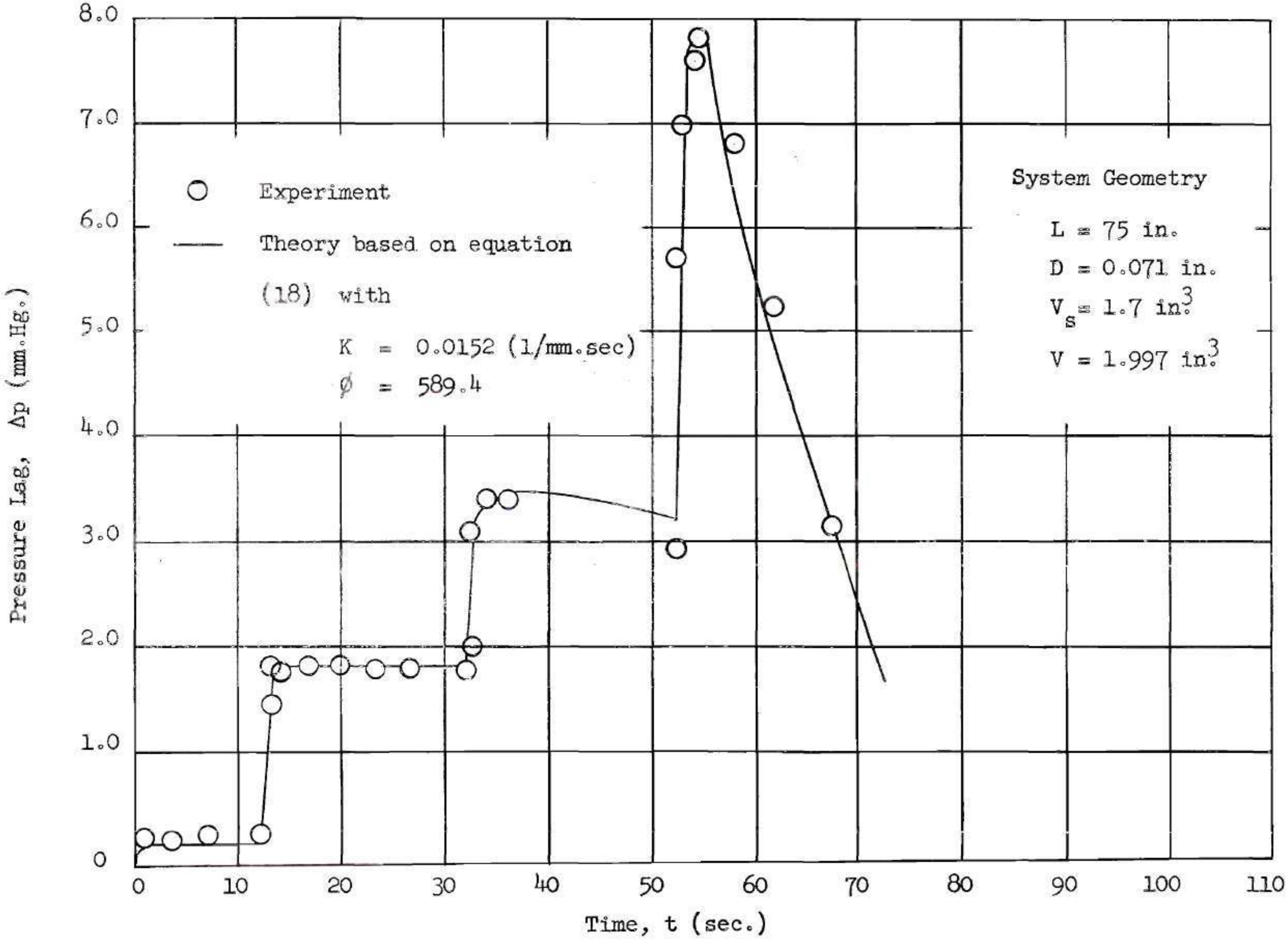



Fig. 4 Correlation of Experiment with Theory for Input Pressure Function No. 1

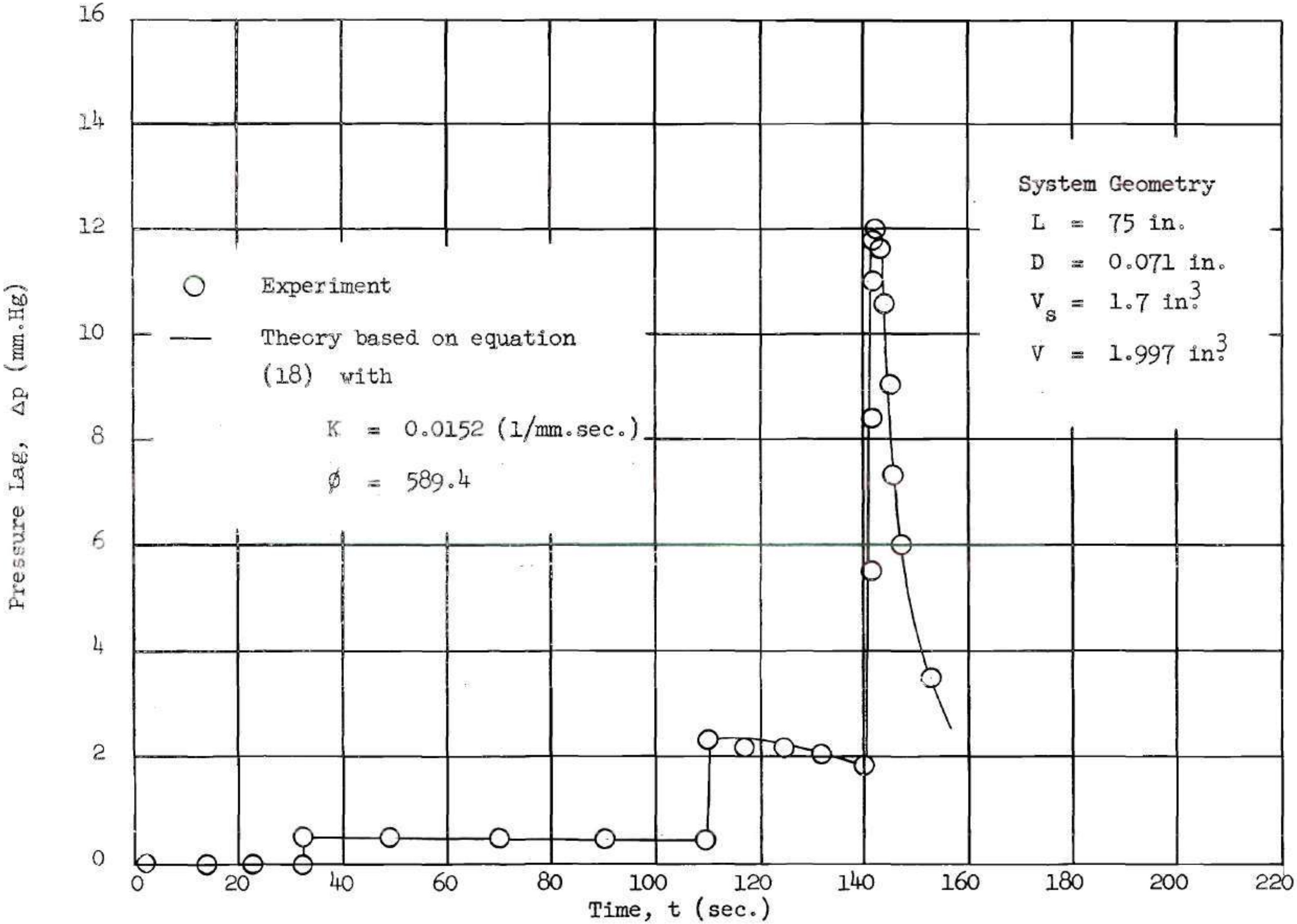



Fig. 5 Correlation of Experiment with Theory for Input Pressure Function No. 4

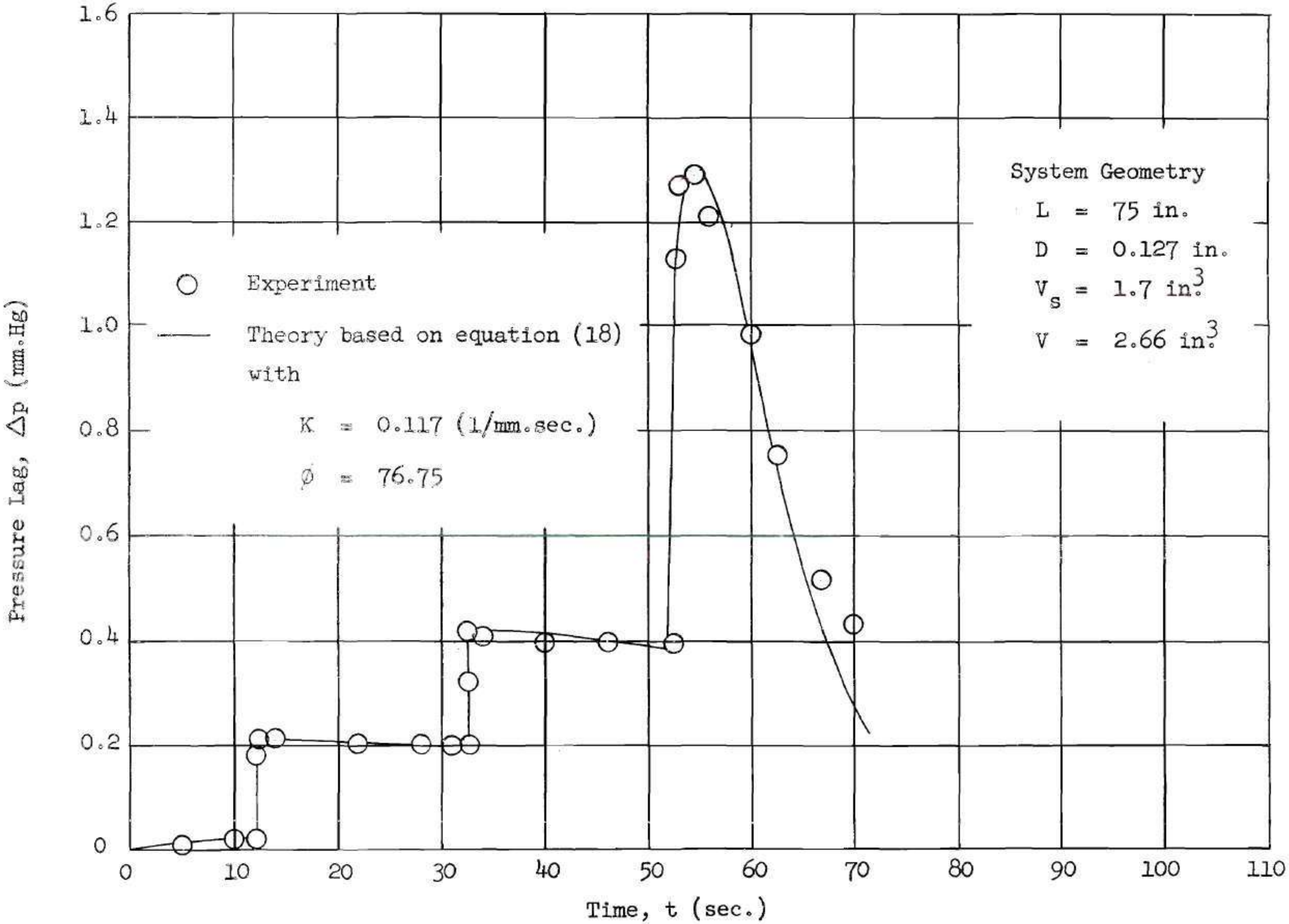



Fig. 6 Correlation of Experiment with Theory for Input Pressure Function No. 1

For the remainder of the runs an inverse method was used. From the experimental values of $p_i$ and $\Delta p$ for a given run (i.e. a given system subjected to a particular pressure input) $K$ was computed from equation (18) at randomly selected values of time. The values of $K$ so determined were then averaged. Fig. 7 is a plot of both empirical and theoretical values of $K$ against the geometric parameter $\phi$. This comparison indicates that solutions of equation (18) for the other runs may be expected to show as good agreement with the experimental data as is demonstrated in Figs. 3, 4, 5, 6.

Validity of Assumptions of Fully-developed, Laminar Flow.--The agreement between the theory of this paper and Kowalsky's experimental data indicates that the assumptions of fully-developed, laminar flow over the entire length of tubing are reasonable for the system geometries tested by Kowalsky. It is of interest, however, to note that the maximum Reynolds number occurring in Kowalsky's tests was found to be of the order of 50, which is well below the lower critical value for pipe flow of approximately 2,000.

Also, the lowest value of $L/D$ for Kowalsky's systems was approximately 190. For lower values of $L/D$ it may be expected that the assumption of fully developed flow will lead to inaccuracies as the portion of the flow which is not fully developed becomes relatively larger.

This effect should show up as a decrease in the value of $K$ since the relatively greater extent of undeveloped flow will cause an increase in the apparent friction factor for the flow in the tube.

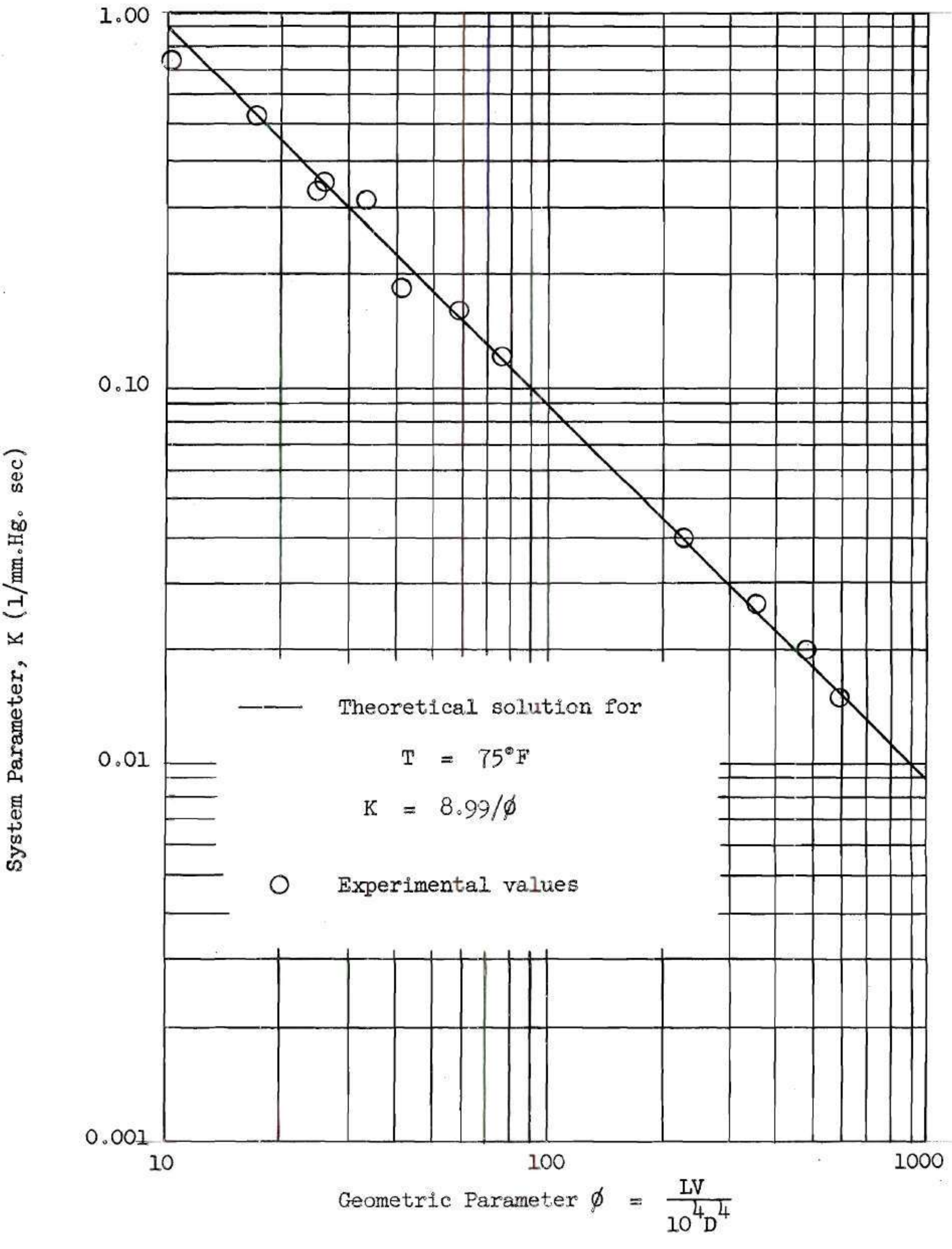



Fig. 7 Experimental and Theoretical Values of K

# CHAPTER IV

## CONCLUSIONS

The results of this investigation may be summarized as follows. For the range of system geometries and pressure input functions tested by Kowalsky the theory provides an accurate method for predicting the response of missile pressure sensing instrumentation to inputs analogous to those experienced by a multi-stage missile in ascending flight. Further, since the input functions employed by Kowalsky have a number of discontinuities in the slope, it is to be expected that equally good agreement would be obtained for input functions with a continuous derivative.

The geometry parameters of Kowalsky's systems lie in the ranges listed below.

| Parameter | Range |
|---|---|
| $L$ | 30" - 75" |
| $D$ | 1/16" - 5/32" |
| $L/D$ | 190 - 1,200 |
| $\phi = LV/10^4D^4$ | 10 - 600 |
| $V_t/V$ | 0.05 - 0.5 |

(The significance of the parameter $V_t/V$ is shown in Appendix A.) The ambient temperature during Kowalsky's tests varied little from 75°F.

# CHAPTER V

## RECOMMENDATIONS

The author would recommend that experimental studies similar to those made by Kowalsky be made on systems with values of $\phi$ outside the range covered by Kowalsky. It is believed that the tubing lengths and diameters tested by Kowalsky cover the ranges which would be found in any practical installation. On the other hand, much larger sensing volumes are quite common, so that emphasis should be placed on systems having larger values of $\phi$.

It would also be of interest to study the effect of decrease in the $L/D$ ratio in order to establish the limit of validity of the assumption of fully developed flow over the entire length of tubing.

Further, tests should be made using different types of pressure input function. This would have bearing primarily on the assumptions of quasi-steady and laminar flow.

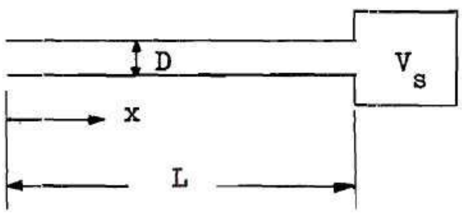
In addition, a study should be made to determine the effect of the presence in the system of connector fittings such as elbows, tees, etc.

## APPENDIX A

Calculation of Mean Pressure in System.--In order to justify replacing $\frac{d\bar{p}}{dt}$ by $\frac{dp_r}{dt}$ in equation (14), the mean pressure in the system $\bar{p}$ will be evaluated from equation (12) which is

$$\bar{p} = \frac{1}{V}\int p\,dV \tag{12}$$

For convenience, Figure 1 is repeated below.




Let $V_s$ be the volume of the sensing element

$V_t$ be the volume of the tubing

$V$ be the total volume of the system $= V_s + V_t$

The pressure distribution in the system is assumed to be as follows. In the tubing the pressure $p$ at a distance $x$ from the input end is given by equation (8), viz.

$$p^2 = (p_r^2 - p_i^2)\,\frac{x}{L} + p_i^2 \tag{8}$$

where $p_i$ is the input pressure $(x = 0)$

and $p_r$ is the response pressure $(x = L)$

In the sensing volume the pressure is equal to the response pressure, $p_r$, throughout.

Then equation (12)

$$\bar{p} = \frac{1}{V}\int p dV \tag{12}$$

becomes

$$\begin{aligned}
\bar{p} &= \frac{1}{V}\left\{ \int_{V_t} p dV_t + p_r V_s \right\} \\
&= \frac{1}{V}\left\{ V_t \int_0^1 p d(\tfrac{x}{L}) + p_r V_s \right\} \\
&= \frac{1}{V}\left\{ V_t \int_0^1 \left[ (p_r^2 - p_i^2)\frac{x}{L} + p_i^2 \right]^{1/2} d(\tfrac{x}{L}) + p_r V_s \right\} \\
&= \frac{1}{V}\left\{ V_t \frac{2}{3}\frac{p_r^3 - p_i^3}{p_r^2 - p_i^2} + p_r V_s \right\} \\
&= \frac{1}{V}\left\{ p_r(V_s + V_t) - \frac{1}{3}V_t(2p_r - 2p_i - p_r + \frac{2p_r p_i}{p_r + p_i}) \right\} \\
&= p_r - \frac{1}{3}\frac{V_t}{V}\Delta p(2 - \frac{p_r}{p_r + p_i}) \\
&= p_r - \frac{1}{3}\frac{V_t}{V}\Delta p(1 + \frac{p_r}{p_r + p_i})
\end{aligned}$$

so that

$$p_r - \bar{p} = \frac{1}{3}\frac{V_t}{V}\Delta p(1 + \frac{p_i}{p_r + p_i}) \tag{A.1}$$

Now, for a missile in ascending flight,

$$p_i \leq p_r$$

so that equation (A.1) implies that

$$p_r - \bar{p} \leq \frac{1}{2}\frac{V_t}{V}\Delta p \tag{A.2}$$

The systems tested by Kowalsky were such that the values of $V_t/V$ lay in the range 0.05 to 0.5. Thus, for systems in which $V_t/V$ is small, the substitution of $\frac{dp_r}{dt}$ for $\frac{d\bar{p}}{dt}$ in equation (14) appears to be justified. For systems wherein $V_t/V$ is of the order of 0.5 (i.e. such that $V_s \approx V_t$) the inequality (A.2) can be replaced by

$$p_r - \bar{p} \leq \frac{\Delta p}{4}$$

or

$$\frac{p_r - \bar{p}}{p_r} \leq \frac{1}{4}\frac{\Delta p}{p_r} \tag{A.3}$$

In this case it is difficult to draw any definite conclusions. However, it has been demonstrated that the theory agrees well with experiment except towards the end of each run, where $\Delta p$ is becoming comparable in magnitude with $p_r$ . Thus, it appears that the substitution of $\frac{dp_r}{dt}$ for $\frac{d\bar{p}}{dt}$ is reasonable except at the end of each run.

## APPENDIX B

The response pressure equation (17) is

$$\frac{dp_r}{dt} = K\,(p_i^2 - p_r^2) \tag{B.1}$$

This equation is a particular case of the generalized Riccati equation[8]

$$\frac{dy}{dt} = P + Qy + Ry^2 \tag{B.2}$$

where P,Q and R are functions of $t$ . Equation (B.2) can be reduced to the canonical form of Riccati's equation[9]

$$\frac{du}{dt} = F(t) + u^2 \tag{B.3}$$

For equation (B.1) the reduction is effected by means of the substitution

$$y = -\,Kp_r \tag{B.4}$$

Thus

$$\begin{aligned}\frac{dy}{dt} &= -\,K\,\frac{dp_r}{dt}\\ &= -\,K^2p_i^2 + K^2p_r^2\\ &= -\,K^2p_i^2 + y^2\end{aligned}$$

Replacing $-K^2p_i^2$ by $F(t)$ yields

$$\frac{dy}{dt} = F(t) + y^2 \tag{B.5}$$

which is clearly of the form (B.3).

The following theorem[9] is of interest. If in equation (B.5) $F(t)$ is a polynomial in $t$ of even degree then no polynomials other than $y = \pm \left[\sqrt{- F(t)}\right]$ can be solutions of equation (B.5). By the symbol $\left[\sqrt{P(t)}\right]$, where $P(t)$ is a polynomial of even degree, is meant the polynomial part of the expansion of $\sqrt{P(t)}$ in a series of descending integral powers of $t$. If $F(t)$ is a polynomial of odd degree, then no polynomial solution of equation (B.5) exists.

Now $F(t) = - K^2 p_i^2$ so that if $p_i$ is a polynomial in $t$, then $F(t)$ will be a polynomial of even degree and the two possible solutions indicated in the above theorem are $y = \pm Kp_i$ which are obviously not solutions of equation (B.1) for $p_i$ a polynomial in $t$ unless $p_i$ is a constant. Thus, solutions of equation (B.1) for $p_i$ a polynomial in $t$ are not themselves polynomials in $t$ unless $p_i$ is a constant.

The potential importance of the above theorem lay in the fact that if one particular solution of equation (B.2) is known, then the general solution can be found by a process involving two integrations.[8] Further, if two particular solutions are known then the general solution can be found by means of one integration.

If no particular solutions can be found the following approach may be of value. It can be shown[9] that the generalized Riccati equation (B.2) is completely equivalent to the linear equation of the second order

$$p \frac{d^2u}{dt^2} + q \frac{du}{dt} + ru = 0 \tag{B.6}$$

For equation (B.5) the reduction can be effected by means of the substitution

$$y = -\frac{1}{u}\frac{du}{dt} \tag{B.7}$$

Then equation (B.5) becomes

$$\begin{aligned} F(t) + y^2 &= \frac{dy}{dt} \\ &= -\frac{1}{u}\frac{d^2u}{dt^2} + \frac{1}{u^2}\left(\frac{du}{dt}\right)^2 \end{aligned}$$

so that

$$\frac{1}{u}\frac{d^2u}{dt^2} + F(t) = 0$$

or

$$\frac{d^2u}{dt^2} + F(t)u = 0 \tag{B.8}$$

The particular advantage in using equation (B.8) instead of equation (B.1) lies in the fact that if $F(t)$ is a polynomial in $t$, then a series solution can be obtained.

For a restricted class of functions $F(t)$, equation (B.5) can be solved in closed form. These functions are of the form $F(t) = ct^m$ where $c$ is a constant and $m$ is of the form $-\frac{4s}{s+1}$ where $s = 0$, 1,2, ... The method of solution is rather involved but may be found, for example, in Reference 10.

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